CHALMERS
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Instantons and Automorphic Representations

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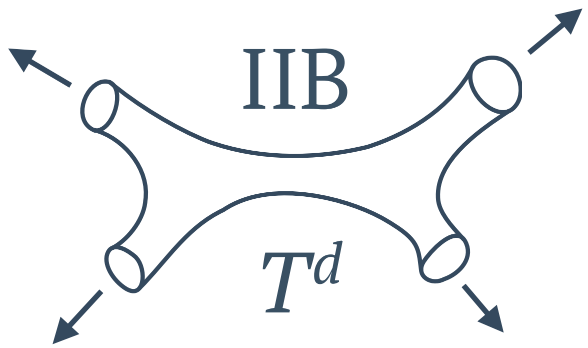
arXiv:1412.5625

[HG, Axel Kleinschmidt & Daniel Persson]

Automorphic forms



$$S = \int R + f_1 \mathcal{R}^4 + f_2 \partial^4 \mathcal{R}^4 + f_3 \partial^6 \mathcal{R}^4 + \ldots$$

[Green-Schwarz, ...]

f_1, f_2, f_3 functions on the moduli space $G(\mathbb{R})/K$

invariant under the U-duality group $G(\mathbb{Z})$

Known explicitly

Automorphic forms

Bao, Basu, Bossard, Cederwall, Fleig, Green, Gubay, Gutperle, Kazhdan, Kiritsis, Kleinschmidt, Lambert, Miller, Nilsson, Obers, Persson, Pioline, Russo, Sethi, Vanhove, Verschinin, Waldron, West, ...

Automorphic representations

$$S = \int R + f_1 \mathcal{R}^4 + f_2 \partial^4 \mathcal{R}^4 + f_3 \partial^6 \mathcal{R}^4 + \dots$$

SUSY constraints $\longrightarrow$ few non-vanishing corrections

$\downarrow$ Differential equations

f_1, f_2, f_3 in small automorphic representations

[Ginzburg-Rallis-Soudry, Green-Sethi, Green-Miller-Vanhove, Pioline, Bossard-Kleinschmidt]

Fourier coefficients

Ten dimensions

$\tau = \chi + ie^{-\phi} \in SL(2,\mathbb{R})/SO(2,\mathbb{R})$

$SL(2,\mathbb{Z})$ invariance $\longrightarrow$ periodic in $\chi = \mathrm{Re}\,\tau$

Fourier expansion

$$f_1(\tau) = Ag_s^{-\frac{3}{2}} + Bg_s^{\frac{1}{2}} +$$

Instanton charge

Perturbative corrections

Instanton measure

SUM OVER DIVISORS

[Green-Gutperle]

Fourier coefficients

For lower dimensions (larger groups)

Instanton charges

$$F_{\vec{m}}[f] = \int\limits_{\substack{\text{unipotent} \\ \text{subgroup of } G}} du\, f(ug)\, e^{2\pi i \langle \vec{m} | u \rangle}$$

Axions

Difficult to compute

Only known for f_1, f_2, f_3 in certain cases

Results

arXiv:1412.5625

[HG, Axel Kleinschmidt & Daniel Persson]

Method for computing Fourier coefficients

of automorphic forms in small representations

by organizing instanton charges into nilpotent orbits.

Key advantage: singles out the few non-vanishing corrections

Builds on recent work by Miller-Sahi and Ginzburg,
using theorems from Matumoto and Mœglin-Waldspurger

Outlook

$$S = \int R + f_1 \mathcal{R}^4 + f_2 \partial^4 \mathcal{R}^4 + f_3 \partial^6 \mathcal{R}^4 + \dots$$

- Apply method to compute instanton effects for $D = 5, 4$ and 3.

  $G = E_6, E_7, E_8$

  $F_{\vec{m}}[f_i]$

- Extend mathematical framework for Kac-Moody groups.

  $G = E_9, E_{10}, E_{11}$

  $D < 3$

Work in progress

[Olof Ahlén, Philipp Fleig, HG, Axel Kleinschmidt & Daniel Persson]

Thank you!

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